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A non-supersymmetric deformation of the AdS/CFT correspondence

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Abstract: We deform the AdS/CFT Correspondence by the inclusion of a non-supersymmetric scalar mass operator. We discuss the behaviour of the dual 5 dimensional supergravity field then lift the full solution to 10 dimensions. Brane probing the resulting background reveals a potential consistent with the operator we wished to insert.

Keywords: AdS-CFT and dS-CFT Correspondence, Supersymmetry Breaking.

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Contents

1. Introduction

A corner stone of the AdS/CFT Correspondence [1, 2, 3] is that the supergravity fields in $AdS_5 \times S^5$ act as source terms for operators in the $\mathcal{N} = 4$ gauge theory dual. It is essential that there is a one to one mapping between the supergravity fields and these operators. A corollary of this relationship is that we can study any deformation of the $\mathcal{N} = 4$ gauge theory since we can introduce all possible operators.

An industry has grown in attempting to generate gravity duals of all interesting deformations of the AdS/CFT Correspondence [4]–[8] and understand how different field theory phenomena are encoded gravitationally. Much of the early work in this respect has concentrated on supersymmetric deformations such as the $\mathcal{N} = 4$ theory on moduli space [9], $\mathcal{N} = 1$ Leigh Strassler theory [10, 11], the $\mathcal{N} = 2^*$ [12]–[16] and $\mathcal{N} = 1^*$ theories [17, 18, 10]. Allowing non-trivial solutions for the supergravity fields is relatively straightforward at the 5d supergravity level. Connecting the resulting backgrounds to the field theory has proven difficult with more success being obtained in the cases where the solutions have been lifted to 10d. The technology to lift solutions to 10d [19, 9, 12, 10] has been developed but in more complicated cases such as the $\mathcal{N} = 1^*$ theory [10] the complexity can grow to prohibit a full solution being found. The major benefit of a 10d solution is that brane probing [1, 15, 16, 11, 20, 21, 22] can be used to directly convert the space-time background into a U(1) gauge theory on the surface of the probe. In this way the gauge coupling, operator parametrization and potentials can be successfully investigated. It should be noted that, since the $\mathcal{N} = 4$ gauge theory is strongly coupled in the ultra-violet, fields can never be decoupled from the strong interactions by making them massive in these models.

More recently interest has turned to non-supersymmetric deformations of gauge/gravity duals [6, 23, 24, 25, 26]. It is interesting to see whether the dualities continue to make sense without supersymmetry and there is also the potential to investigate new phenomena not present in supersymmetry. Apart from an early paper on non-supersymmetric defor-

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mations in 5d [6], recent attention [23]–[26] has focused on deformations of more involved $\mathcal{N}=1$ supersymmetric constructions such as the Maldecena Nunez [27] and the Klebanov Strassler [28] backgrounds. These theories have discrete vacua and hence supersymmetry breaking perturbations will not result in an unstable background. In this paper we return to deforming the orginal AdS/CFT correspondence. We will in fact introduce a mass term of the form $(\phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2-2\phi_5^2-2\phi_6^2)$ which is naively unbounded. Our interest is in developing the technology to find and lift these solutions to 10d so we will not be so concerned by the runaway behaviour (although the 10d solution we provide correctly reproduces the expected behaviour). One might hope that there would be such backgrounds that are really stable since an $\mathrm{SO}(6)_R$ singlet scalar mass term is not visible in the supergravity solution as it is not in a short multiplet. It's presence could stabilize the solution. Note that the supersymmetric deformations [5, 12, 10] already mentioned require this operator to be present. In fact our brane probe potential reveals the operator not to be present in our 10d lifts. Our solution is also of interest since it is probably the simplest example of a non-supersymmetric deformation; only the metric and four potential fields are non-zero.

In the next section we will discuss the introduction of our deformation at the 5d supergravity level. In section 3 we then lift the full solution to 10d, although one function in the four form is only found numerically. In section 4 we brane probe the background with a D3 brane and show that asymptotically the background indeed includes the operator we hoped to introduce showing the consistency of the techniques. Finally we plot the potential seen by the probe for the full solution.

2. Deformations in 5d supergravity

According to the standard AdS/CFT Correspondence map [2, 3] each supergravity field plays the role of a source in the dual field theory. The simplist possibility is to consider non-trivial dynamics for a scalar field in the 5d supergravity theory. We only allow the scalar to vary in the radial direction in AdS with the usual interpretation that this corresponds to renormalization group running of the source. As is standard in the literature [4, 6] we look for solutions where the metric is described by

$$ds^2 = e^{2A(r)}dx^\mu dx_\mu + dr^2 \tag{2.1}$$

where $\mu = 0..3$ and r is the radial direction in AdS_5. The scalar field has a lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\lambda)^2 + V(\lambda)\,. \tag{2.2}$$

There are two independent, non-zero, elements of the Einstein tensor (G_{00} and G_{rr}) giving two equations of motion plus there is the usual equation of motion for the scalar field [4]

$$\begin{aligned} \lambda'' + 4A'\lambda' &= \frac{\partial V}{\partial\lambda} &(2.3)\\ 6A'^2 &= \lambda'^2 - 2V &(2.4)\\ -3A'' - 6A'^2 &= \lambda'^2 + 2V\,. &(2.5) \end{aligned}$$

In fact only two of these equations are independent but it will be useful to keep track of all of them.

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In the large r limit, where the solution will return to AdS$_5$ at first order and $\lambda \to 0$ and $V \to m^2\lambda^2$, only the first equation survives with solution

$$\lambda = \mathcal{A}e^{-\Delta r} + \mathcal{B}e^{-(4-\Delta)r} \tag{2.6}$$

with

$$m^2 = \Delta(\Delta - 4) \tag{2.7}$$

$\mathcal{A}$ is interpreted as a source for an operator and $\mathcal{B}$ as the vev of that operator since e^r has conformal dimension 1.

If the solution retains some supersymmetry then the potential can be written in terms of a superpotential [7]

$$V = \frac{1}{8}\left|\frac{\partial W}{\partial \lambda}\right|^2 - \frac{1}{3}|W|^2 \tag{2.8}$$

and the second order equations reduce to first order

$$\lambda' = \frac{1}{2}\frac{\partial W}{\partial \lambda}, \qquad A' = -\frac{1}{3}W\,. \tag{2.9}$$

2.1 A scalar operator

Let us now make a particular choice for the scalar field we will consider. We take a scalar from the multiplet in the 20 of SO(6). These operators have been identified [3] as playing the role of source and vev for the scalar operator $tr\phi_i\phi_j$ in the field theory. In particular we will chose the scalar corresponding to the operator

$$\mathcal{O} = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 - 2\phi_5^2 - 2\phi_6^2\,. \tag{2.10}$$

This scalar has been studied in the literature [9, 20] already in its role of describing an $\mathcal{N} = 4$ preserving scalar vev and as a mixture of a mass term and a vev in the $\mathcal{N} = 2^*$ gauge theory [12, 13]. The potential for the scalar, which we will write as $\rho = e^{\lambda/\sqrt{6}}$ is given by

$$V = -\frac{1}{\rho^4} - 2\rho^2 \tag{2.11}$$

and the three equations of motion become

$$\frac{\rho''}{\rho} - \left(\frac{\rho'}{\rho}\right)^2 + 4\frac{\rho'}{\rho}A' = \frac{\rho}{6}\frac{\partial V}{\partial \rho} \tag{2.12}$$

$$6A'^2 - 6\left(\frac{\rho'}{\rho}\right)^2 = -2V \tag{2.13}$$

$$A'' = -4\left(\frac{\rho'}{\rho}\right)^2. \tag{2.14}$$

The last of these is the sum of (2.4) and (2.5). The asymptotic ($r \to \infty$) solutions take the form

$$\lambda = \mathcal{A}e^{-2r} + \mathcal{B}re^{-2r} \tag{2.15}$$

with $\mathcal{A}$ the scalar vev and $\mathcal{B}$ a mass term for the operator $\mathcal{O}$.

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In the special case where only the first part of the solution is present the deformation preserves $\mathcal{N}=4$ supersymmetry. The superpotential is

$$W=-\frac{1}{\rho^2}-\frac{1}{2}\rho^4 \tag{2.16}$$

and the second order equations reduce to the first order equations

$$\frac{\partial\rho}{\partial r}=\frac{1}{3}\left(\frac{1}{\rho}-\rho^5\right), \qquad \frac{\partial A}{\partial r}=\frac{2}{3}\left(\frac{1}{\rho^2}+\frac{1}{2}\rho^4\right) \tag{2.17}$$

with solution [9]

$$e^{2A}=l^2\frac{\rho^4}{\rho^6-1} \tag{2.18}$$

with l^2 a constant of integration.

2.2 Non-supersymmetric first order equations

In [29] it was pointed out that using Hamilton Jacobi theory the second order equations could be replaced by a system of first order equations. They further stated that a "superpotential", W, could be found which resulted in the equations (2.9) even for the non supersymmetric solution with only $\mathcal{B}$ switched on. A similar result was obtained in [30, 31] but as a requirement for the RG flow solution to be stable. Further analysis along these lines can be found in [32, 33]. Reducing the equations to first order would be very helpful, but the system we discuss here can not be.

Consider the UV of the theory where, expanding (2.11)

$$V=-3-2\lambda^2+\sqrt{\frac{8}{27}}\lambda^3+\ldots \tag{2.19}$$

we can attempt to find a superpotential W that reproduces this potential via the trial form

$$W=a+b\lambda^2+c\lambda^3+\ldots. \tag{2.20}$$

Working to quadratic order one finds

$$a=-3, \qquad b=-2. \tag{2.21}$$

The solution for b comes from a quadratic equation with degenerate roots hinting at the two forms of the solution. However, it is then easy to show that at higher orders there is a unique series (eg $c=\sqrt{2/27}$) and it is simply the supersymmetric solution. We have therefore not been able to find a superpotential that describes the non-supersymmetric solution and are forced to numerically solve the second order equations. Of course our geometry is intrinsically unstable since we have introduced an unbounded operator in the field theory. Apparently the stability of the flow is essential for the system to reduce to first order.

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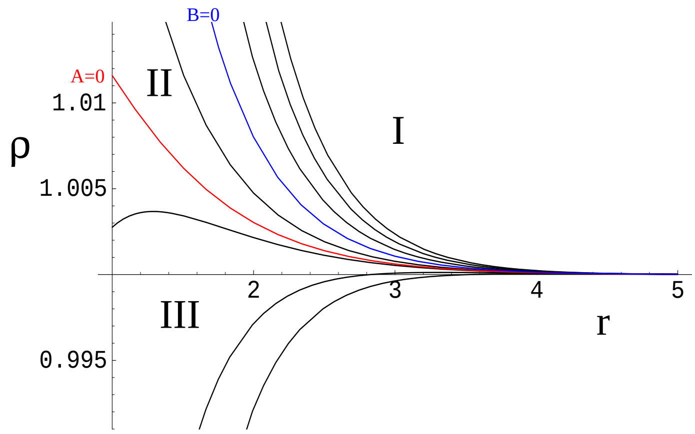

Figure 1: Plots of ρ vs r for a variety of initial conditions on ρ'. The vev only initial condition solution is marked with $\mathcal{B} = 0$ and mass only initial condition with $\mathcal{A} = 0$. The marked regions are explained in (2.22).

2.3 Numerical solutions

The second order equations of motion are easily solved. In figure 1 we show the numerical behaviour of ρ. For this plot we fix $\rho(r = \Lambda_{UV})$ and vary the derivative. The purely vev supersymmetric solution ($\mathcal{B} = 0$) and purely masslike case ($\mathcal{A} = 0$) are labelled. The three regions (bounded by the $\mathcal{A} = 0$ and $\mathcal{B} = 0$ curves) correspond to

$$\begin{array}{ccc} & \mathcal{A} & \mathcal{B} \\ I & +ve & -ve \\ II & +ve & +ve \\ III & -ve & +ve \end{array} \tag{2.22}$$

In all these cases the function $A(r)$ only deviate from each other and $A(r) \sim r$ by a small amount so a plot is unrevealing. Note that most of these solutions become singular before $r = 0$. When lifted to 10d this singular point is expected [9] to correspond to the position of the D3 brane sources in the background. For most of these solutions there is a scalar vev and so the D3 branes are expected to have moved away from the origin. The mass only solution ($\mathcal{A} = 0$) on the other hand can be extended to $r = 0$ which is consistent with the D3 branes being pinned at the origin.

It has proven difficult to extract aspects of the field theory from the 5d supergravity backgrounds. More success has been had at the 10d level where techniques such as brane probing can be used to connect to the field theory. We shall therefore move to discussing the lift of these solutions to 10d in the next section.

3. The 10d background

To lift the 5d solution to 10d requires the procedure outlined in [19]. Finding the metric is complicated but we will be able to short cut the process since the lift of the 5d solution where the $\mathcal{N} = 4$ theory is on moduli space has already been written down. In particular

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the solution where our scalar corresponds to a vev has been studied in [9, 20] (it is also the limit of the metrics in [12, 13, 10] with some of the fields switched off). That solution is given by

$$ds^2 = \frac{X^{1/2}}{\rho} e^{2A(r)} dx_{//}^2 + \frac{X^{1/2}}{\rho} \left(dr^2 + \frac{L^2}{\rho^2} \left[d\theta^2 + \frac{\sin^2\theta}{X} d\phi^2 + \frac{\rho^6 \cos^2\theta}{X} d\Omega_3^2 \right] \right), \tag{3.1}$$

where $d\Omega_3^2$ is the metric on a 3-sphere and

$$X \equiv \cos^2\theta + \rho^6 \sin^2\theta \,. \tag{3.2}$$

For consistency there must also be a non-zero C_4 potential of the form

$$C_4 = \frac{e^{4A} X}{g_s \rho^2} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \,. \tag{3.3}$$

Note that the solution has the same $SO(2) \times SO(4)$ symmetry as our operator (2.10).

Clearly the lift of the full solution of the second order equations has this as a limit. In fact the procedure for finding the form of the metric does not depend on the supersymmetric solution and we may take it over directly to our case. The C_4 potential though will change since the supersymmetric first order equations of motion were used in its derivation [9, 12].

In fact the 10d supergravity equations of motion we must concern ourselves with are relatively few [12] since only the metric and C_4 are non-zero. There are the Einstein equations

$$R_{MN} = T_{MN} = \frac{1}{6} F_N^{PQRS} F_{PQRSM} \tag{3.4}$$

and

$$F_{(5)} = {}^* F_{(5)} \,, \qquad dF_{(5)} = 0 \,. \tag{3.5}$$

The self duality condition can be imposed by using the ansatz

$$F_{(5)} = \mathcal{F} + {}^* \mathcal{F} \,, \qquad \mathcal{F} = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dw \tag{3.6}$$

where $w(r, \theta)$ is an arbitrary function.

There are three independent non-zero elements of R_{MN} which factorize into the useful equations

$$R_0^0 - R_r^r = \frac{1}{2} g^{00} g^{11} g^{22} g^{33} g^{rr} \left(\frac{\partial w}{\partial r} \right)^2 \tag{3.7}$$

$$R_0^0 + R_r^r = \frac{1}{2} g^{00} g^{11} g^{22} g^{33} g^{\theta\theta} \left(\frac{\partial w}{\partial \theta} \right)^2 \tag{3.8}$$

$$R_\theta^r = \frac{1}{2} g^{11} g^{22} g^{33} g^{44} g^{rr} \left(\frac{\partial w}{\partial \theta} \frac{\partial w}{\partial r} \right) . \tag{3.9}$$

The right hand side of these equations are straightforward but laborious to explicitly calculate. The resulting output is lengthy but can be simplified by using the second order

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equations of motion to eliminate ρ'', A'' and A'^2. The resulting background will therefore reproduce the full second order equations of motion. We find

$$R^0_0 - R^r_r = -\frac{18\sin^2\theta\cos^2\theta\rho^5\rho'^2}{X^{5/2}} \tag{3.10}$$

$$R^0_0 + R^r_r = -\frac{(2\cos^2\theta - (\cos 2\theta - 3)\rho^6)^2}{2\rho^3 X^{5/2}} \tag{3.11}$$

$$R^r_\theta = -\frac{3\sin^2\theta\rho'((\cos 2\theta - 3)\rho^6 - 2\cos^2\theta)}{2X^{5/2}} \tag{3.12}$$

(3.7) thus reduces to

$$\left(\frac{\partial w}{\partial \theta}\right) = \frac{6e^{4A}\cos\theta\sin\theta\rho'}{\rho} \tag{3.13}$$

which can be directly integrated and w put in the form

$$w(r,\theta) = \frac{e^{4A}}{\rho^2} - \frac{3\sin^2\theta\rho' e^{4A}}{\rho} - e^{4A}F(r) \tag{3.14}$$

where $F(r)$ is as yet undetermined.

Note that the supersymmetric limit corresponds to $F(r) = 0$ and ρ' replaced using the supersymmetric first order equation of motion (2.17). We should not be surprised that derivatives of ρ enter directly into the solution since introducing a mass term corresponds explicitly to introducing an extra degree of freedom via precisely this derivative.

F can then be found using either of the other two equations (the third equation providing a check on the consistency of the solution). It is the solution of

$$-2 - 2\rho^6 = -4\rho^2 A' + 4\rho^4 F A' + \rho^4 F' + 2\rho\rho'. \tag{3.15}$$

We have not been able to solve this equation explicitly but in the UV limit the solution takes the form

$$F = \frac{1}{3}\left(\frac{1}{\rho} - \rho^5\right) - \rho' + \dots. \tag{3.16}$$

which clearly vanishes in the supersymmetric limit given (2.17). For a general numerical solution of the second order equations of motion we can set the boundary conditions on F using this asymptotic form and hence find F numerically for all r.

The solution then faces its strongest test since $F_{(5)}$ must also satisfy its bianchi identity (3.5). At first sight this appears to be a challenge; since w contains a derivative of ρ the bianchi identity is a third order equation.

The Bianchi identity (3.5) is (since $d\mathcal{F} = 0$)

$$dF = \left(\partial_r\left[\sqrt{g}g^{00}g^{11}g^{22}g^{33}g^{rr}\partial_r\omega\right] + \partial_\theta\left[\sqrt{g}g^{00}g^{11}g^{22}g^{33}g^{\theta\theta}\partial_\theta\omega\right]\right) d\Omega_5 \wedge dr = 0\,. \tag{3.17}$$

Hence we must check that

$$\partial_r\left[X^2 e^{-4A}\rho^4\sin\theta\cos^3\theta\sqrt{\det S_3}\partial_r\omega\right] + \partial_\theta\left[X^2 e^{-4A}\rho^6\sin\theta\cos^3\theta\sqrt{\det S_3}\partial_\theta\omega\right] = 0\,. \tag{3.18}$$

In fact explicit computation, using the second order equations of motion and (3.15), shows that this third order equation is satisfied and the solution survives.

Given the complete numerical 10d lift of our non-supersymmetric solutions we can study the background for signals that it correctly encodes the field theory dynamics.

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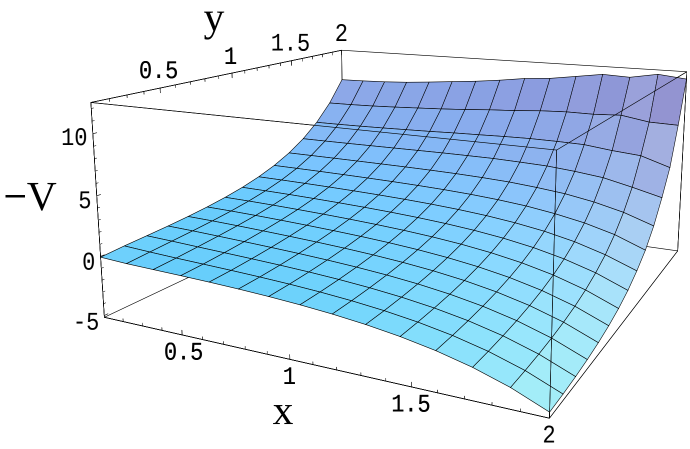

Figure 2: The probe potential plotted over the $r-\theta$ plane for the mass only case ($\mathcal{A}=0$).

4. Brane probe potential

The most succesful technique for connecting backgrounds and their dual field theories has been brane probing [1, 15, 16, 11, 20, 21, 22] which converts the background to the U(1) theory on the probe's surface. We thus substitute the background into the Born-Infeld action

$$S_{probe} = -\tau_3 \int_{\mathcal{M}_4} d^4x \det \left[G_{ab}^{(E)} + 2\pi\alpha' e^{-\Phi/2} F_{ab} \right]^{1/2} + \mu_3 \int_{\mathcal{M}_4} C_4 \,, \tag{4.1}$$

The resulting scalar potential is given by

$$V_{probe} = -e^{4A} \left[\frac{X}{\rho^2} + \frac{3\sin^2\theta\rho'}{\rho} - \frac{1}{\rho^2} + F \right]. \tag{4.2}$$

It is illuminating to evaluate this potential at leading order in the UV with

$$\rho = 1 + ve^{-2r} + m^2 re^{-2r} + \dots \,. \tag{4.3}$$

We find

$$V = m^2 e^{2r}(2 - 6\sin^2\theta) + \dots \,. \tag{4.4}$$

The scalar vev vanishes from the potential at this order consistent with the existence of the $\mathcal{N}=4$ moduli space. The mass term reproduces precisely the mass operator we expected in (2.10) remembering that e^r plays the role of a scalar field. We conclude that the 10d background shows all the correct behaviour to be dual to the non-supersymmetric gauge theory with scalar masses.

Finally we numerically find the full solution for $A(r)$, $\rho(r)$ and $F(r)$ for the mass only boundary conditions ($\mathcal{A}=0$) using (2.3), (2.4), (2.5) and (3.15), (3.16). We then plot the full solution for the probe potential in the $r-\theta$ plane for the mass only solution ($\mathcal{A}=0$) in figure 2. The plot fits well with the claim that the mass operator (2.10) is present. The supersymmetric solutions ($\mathcal{B}=0$) give a flat probe potential. Other non-supersymmetric solutions reproduce the form of figure 2 upto a sign change dependent on the sign of $\mathcal{B}$.

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We conclude that we have successfully found the 10d gravity dual of this simple non-supersymmetric deformation of the AdS/CFT Correspondence.

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